Giovanni Battista Amici

Description of various instruments for measuring angles by reflection

(1836)

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In 1822 I published a description of a new instrument for measuring angles, especially at sea, which I called a Prism reflecting sector.

[...] The idea that I had of replacing ordinary mirrors with prisms appeared to fulfil the wishes of observers. My Sector is able to measure angles from zero to 180 degrees or more in an extremely simple fashion, requiring only two rectangular isosceles prisms, one mobile on the alidade that bears the vernier, the other alongside it, fixed to the plane of the graduated limb. Therefore the objects are each seen solely by internal reflection by their respective prisms, unlike the situation with the Sextant where one of the objects is observed directly and the other is observed by double reflection by the mirrors.

[...] Baron von Zach, a most ardent promoter of the fabrication and improvement of prismatic sectors, on being informed that it was impossible to find pure glass in other countries, contacted the Munich workshops that enjoyed the best reputation in that field.

[...] Fraunhofer’s molten glass was, as was to be expected, of a marvellous transparency and homogeneity. I could use it to make prisms that reached a magnification of sixty times with such clarity as to be able to distinguish the double star Castor, which has a separation of 5",2. But the Bavarian optician’s reply that he was unwilling to sell a precious material that he alone was capable of making forced me to suspend my work.

[...] My research then turned to another principle, namely the common one of mirrors, from the various arrangements of which I hoped to achieve some improvements. Indeed I found that just two mirrors positioned over a whole circle instead of a sector, in the manner that I shall explain, offered some of those advantages that I was seeking.

Fig. 2 shows the instrument that I built.

[...] Although the repeating circle is revealed to the public for the first time in this paper, it was actually first built in 1824. During a journey I made in 1827 to France and England, the leading opticians and mechanics in these countries examined it together with other instruments of my own invention that I took with me.

[...] The idea of making two flat mirrors rotate one above the other in order to measure the angles dates from a much earlier period than Troughton. Many writers attribute it to Caleb Smith, who proposed an Octant based on this principle which still bears the inventor’s name.

It is a little known fact that this same Smith had suggested using mirrors or prisms without distinction for his Octant and I too was unaware of it when I described my reflecting sector. Nonetheless, he did not discover the most important property that prisms have of reflecting light even when the plane of reflection is parallel to the incident rays, a property which I believe to have been the first to have noted and recognised as useful for measuring angles in a simple fashion from zero up to 180°.

In truth it is most curious that a period of eleven years should have been a sufficient period of time for some people to have forgotten this, and that a certain Steinheil of Munich should have had a prism circle of his invention built by the same Ertel, whom Baron von Zach had contacted to obtain on my behalf the glass I needed to make the prisms for my Sector.

[...] However, one difference sets my instrument apart from that of Munich. In the latter the prisms are located one above and one below, whereas I proposed positioning them alongside each other because that way they reveal objects twice as distinctly as seen in the reformed modern arrangement. If the two prisms lie one alongside the other, two reflected luminous bands are produced, each with the same width of a quarter of the hypotenuse.

[...] As soon as the prism circle had been produced by the Ertel workshop, Mr. J. G. Horner notified me in a courteous letter dated 20 February 1833. This distinguished Astronomer and Navigator, who was favourable to my Sector and had used it eleven years earlier in Genoa with Baron von Zach, offered to mediate on my behalf to procure clear glass from the large manufactories of Monte Jura. I could hardly have accepted such a kind offer with anything but jubilation, and just a few months later I was already in possession of four large stripe-free Crown-glass prisms from the manufactory of Widow Guinand in Soleure, which was capable of melting Crown glass for other similar and most perfect prisms on request. This fortunate circumstance brought my attention back to the subject to which I had previously applied myself and helped to make a considerable improvement. I am therefore presenting to the public a new prism repeating circle in which I have introduced all the modifications that experience has shown me to be useful.

[...] Fig. 5 shows the instrument. AB is a brass circle with a silver rim of diameter six inches. An alidade NM which can move around the circle with the tangential screw V contains two opposing verniers. The alidade carries in the centre the rectangular isosceles glass prism P which is fastened in that position by four screws, which also serve to make the three edges of the prism parallel to the axis of rotation. The circle together with its alidade rotates above a second concentric axis by means of another tangential screw U attached to the lower frame CDE. The end of outwardly projecting arm C is joined to the base of a second prism Q, which base also supports two series H, L of rings containing the coloured glasses. With the aid of the screw R, the prism Q is positioned with its edges parallel to the edges of P and serves to reflect the light which it receives from prism P onto the telescope S supported by the arm E. The telescope can be moved up or down relative to the plane of the limb and set parallel to it.

From this description it will now be easy to understand how the instrument works. First, I should note that the reflecting face of the prism Q is inclined to the axis of the telescope at a constant angle of 45° and occupies only the lower portion of the objective. So the upper part of the telescope is used to look directly at the distant object, and in the same manner as with the mirrors of known circles or sextants, by rotating prism P the reflected image is brought into the field of view and the images are superimposed or brought into contact. Regardless of the angle indicated by the verniers, the point where the two images, direct and reflected, of the same object coincide is the point where numbering begins, in other words the zero point at which the faces of the prisms are parallel. This parallelism is shown in figure 5; on moving from here to another position by turning the alidade, the sum of the arcs travelled by the two verniers is equal to the angle that a reflected ray would make with a direct ray reaching the eye by the same line, which is equivalent to saying that it is equal to the angular distance of the two observed objects.

Having turned the alidade 45° from N towards A, which is the direction in which the numbers of the divisions proceed, the face opposite the right angle of the prism P is parallel to the incident rays originating from an object situated 90° from the direction of the telescope; and although this is the least favourable position in terms of the quantity of light reflected by the mobile prism, nonetheless there remains sufficient light, consisting of a strip or aperture of about four lines width and of the same height as the prism. But if we continue to move the alidade in the same direction by many more degrees, the luminous strip will subsequently diminish in width and eventually be lost completely. It may therefore appear that this prism can only be used to measure angles of up to 90° or slightly greater, whereas it can in fact be used without any obstacle for measuring angles of up to two right angles with the same precision and distinctness as small angles. Indeed, suffice it to consider that when the vernier N has reached 45°, if the alidade is turned by 180° the plane of reflection remains parallel to itself, while the right angle of the prism assumes a position diametrically opposite to the centre of the circle. So from 90° to 180° reflection continues to take place in the same way as from 90° to zero. Here it should be noted that it is not necessary to make any extra reading or further adjust the instrument in order to turn the alidade by 180°, the only difference being that the names of the verniers are switched round in order to obtain an angle of between 90° and 180° from the sum of their arcs.

Thus far we have assumed that the alidade turns according to the order of numbering, in which case we subsequently view by reflection all the objects that occupy the semi-circumference situated for example on the right of the observer. However, given that the alidade can also move in a retrograde direction, what are the angles that can be determined by this movement on the other side? Anyone who has a knowledge of optics will immediately appreciate that it is possible to perform a rotation of approximately only 6° (depending on the quality of the glass), after which total internal reflection of the prism ceases and is converted into refraction. But the provision I have imagined of tin-plating the large face of the prism overcomes this limit and the rays originating from any object positioned on the observer’s left side undergo the necessary reflection until the incident rays are intercepted by the fixed prism. This does not occur until the eightieth degree. It follows that for these eighty degrees my circle has the property of measuring the angles on either side of the zero point, and is capable of multiplication in the same way as the Borda circles, in other words collimating alternately with the two objects without changing the position of the plane of the instrument, or collimating with the same object and turning the circle around with every observation. In reality, in the Borda circle this operation extends to more than 120°, but twenty or thirty degrees are excluded due to the shadow of the small mirror and the dark glasses. In my circle the entire scale of 80° remains free. Furthermore, another method of repetition can be practised in the circle I am describing and is valid for the entire extension of 180 degrees. This is done by keeping the instrument constantly turned in the same direction with the telescope sighted at one of the objects, and for each repetition starting out with the reflecting faces of the prisms parallel, an arrangement that is recognised by virtue of the superimposition of the two images of the object to which it is collimated. As the telescope S is sighted at an object X (fig. 5), the direct and reflected images of which are superimposed, the alidade N is turned towards A to bring the image of X into contact with that of another object Z, thus generating the simple arc XZ. Then using screw U to make the circle move backward until the two images of X are again superimposed, the previous measurement is repeated by moving the alidade N, thereby creating the double angle, and proceeding in the same fashion to triple and quadruple the angle, etc. As long as the angle that is repeated is no greater than 90°, no particular measures are required to avoid confusion in the readings of the divisions. But when the angle is greater, in which case the alidade must as I said be turned by 180°, with every odd numbered observation the verniers change name and with every even numbered observation they retain their own name. For example, if the vernier N starts out from zero degrees and the angle to be measured is 170°, in the first observation it is the vernier M that will be 85° from zero and in the second observation the vernier N that will indicate the angle of 170°.

[...] The telescope on my circle is built according to the same principles as the new achromatic microscopes, in other words the objective consists of two double-glass objectives positioned in sequence. This results in an aperture of ten lines with a focal length of just four inches, and magnifies five and fifteen times with achromatic eyepieces. Although the objective is made of four lenses, because these lenses are glued together with mastic in couplets and because of the whiteness of the Crown-glass, they absorb a proportionately smaller quantity of light than the triple English objectives which are not glued and are made with green Crown-glass. I still use objectives with just two glasses, of the same aperture of 10 lines and with a focal length of seven inches. Because the magnification of five times in an aperture of ten lines produces the *maximum distinctness*, the emerging beam of light being equal to the greatest width of the pupil, I believe that this will make it easier to read the altitudes of stars at sea without making the image of the star or the horizon too weak.

[...] The graduations of my circle are of 20 seconds, each degree having three parts, and the vernier sixty. I am certain that none of the limb graduations is incorrect by as much as 5", and consequently a greater subdivision would be easy to perform, but a number of considerations led me to prefer the one I made.

[...] As regards the light weight of the instrument, I have succeeded in perfectly eliminating bending of the circle simply by fitting a handle, as shown in fig. 9. This way it remains robust and convenient, weighing 1.43 kilograms ready for observation.

In the *Astronomische Nachrichten* that I mentioned above [nos. 243 and 247 of 1833], we find it written that *it is impossible to build prisms with two equal angles*. This observation prompted a great German astronomer to develop the theory of the so-called prismatic circles of Steinheil, of which I have obtained only the first part, No. 254. The reader will also remember that in his letter to Zach, Fraunhofer talks of the complex and very difficult work of the prisms, which would result in an exorbitant price. So would this stated impossibility, or at least severe difficulty, be a new obstacle to the widespread introduction of a good instrument? I do not believe so. And to defy all opposition, I shall publish here my method of examining prisms in order to evaluate their quality, as privately already shared with Baron von Zach and with other friends of mine. An outline of the method will be sufficient to guide instrument makers in carrying out the work to the utmost perfection.

The principle on which the experiment is based uses the property that an almost isosceles glass prism has of reflecting light even if an incident ray originates from an object located on the opposite side of the reflecting surface. From this property it follows that an object can be seen simultaneously by external reflection from the face adjacent to the two equal angles and by internal reflection from the same face. Observing in this way for example the Sun, if the two discs that derive from the two opposite reflections are exactly superimposed, we have a sure criterion for the perfect equality of the two angles of the prism; namely that if (assuming the axis of the prism to be horizontal and the reflecting face upwards) the internal reflected image of the Sun lies below the image generated by external reflection, then the angle closest to the observer will be smaller than the one further away: vice versa, if the refracted-reflected image is above the one that is simply reflected, this suggests that the smaller angle is the one furthest away. Let ABC in fig. 10 be a prism with base angles c, $c + x$. Let the ray falling on the face CB have an incidence I and let r be the corresponding angle of refraction. Let us call z the internal angle of reflection. Let i be the new incidence and R the angle of emergence. If $m : n$ represents the ratio of the sines of incidence and refraction from air to glass, we will have:

$$\text{sen.}r = \frac{n\,\text{sen.}I}{m},$$

$$z = 90° - \text{Arc. sen.}\,\frac{n\,\text{sen.}I}{m} - c,$$

$$i = \text{Arc. sen.}\,\frac{n\,\text{sen.}I}{m} - x, \text{ but}$$

$$\text{sen.R} = \text{sen.I} \cos.x - \text{sen.}x \sqrt{\frac{m^2}{n^2} - sen.^2 I}$$

This formula serves to calculate the inclination that the incident ray makes with the emerging ray in order to compare it with the inclination that the reflected ray would have with the direct ray if instead of the prism, setting $m = n$, we considered its base AB as a simple plane mirror. This comparison would clearly show the corrections to be made to the angles observed in the instrument using a mobile non-isosceles prism. But without going into detail regarding these applications of the formula, we can immediately see that if the angle I is greater than the angle c, for example by the quantity u, then a ray PS parallel to the one incident above CB, originating from the same point of a distant object, can meet the face AB and reflect externally below the angle u. Now if the prism were perfectly isosceles, the angle I would be equal to the angle R, the ray exiting from inside the prism would be parallel to QS, and therefore the images of the same point seen by simultaneously internal and external reflection would precisely coincide. But in the event that the angles at the base of the prism are unequal, the one on the observer's side being greater than the other by the quantity x, then the reflected ray QS and the ray leaving the prism will both be on the side of the object; indeed the formula shows that I – R is always greater than x. So in this case the object seen by external reflection will appear to be in a lower position than the same object seen by internal reflection, and the opposite will occur when the angle on the observer's side is the smaller one.

No goniometer or theodolite can measure the angles of a prism with the same degree of precision as is achieved with the observation that I have suggested. The prisms made in my workshop pass this extremely stringent test, which makes it unnecessary to take account of corrections when they are mounted on the circle. Furthermore, the previous experiment is also suitable for determining whether the glass is pyramidal in shape rather than a prism, in which case the two different images will appear one alongside the other, and the one reflected internally will be on the same side as the vertex of the pyramid. Perfectly fabricated prisms are only required on the alidade; it is of no particular consequence if the other immobile prisms have two quite different angles, provided that the difference is not such as to produce indistinctness. The immobile prisms can also be substituted by parallel plane mirrors, which produce the same effect. [...]

(English Translation by John Freeman)